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Simultaneous confidence band for nonparametric fixed effects panel data models

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HIGHLIGHTS

- Transforming nonparametric fixed effects panel data model into partially linear model.
- Based on the profile least-squares method, the fixed effects are removed.
- The asymptotic distributions are used to construct SCB of the nonparametric function.
- The bootstrap procedure is proposed to construct SCB of the nonparametric function.
- The proposed methods can be extended to various semiparametric panel data models.

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ABSTRACT

This paper constructs the simultaneous confidence band for the nonparametric function in nonparametric fixed effects panel data models. We first transform the nonparametric fixed effects panel data models into the partially linear models. We then obtain the estimator of the nonparametric function and remove the fixed effects using the profile least-squares method. Finally we apply the established asymptotic results to construct the simultaneous confidence band for the nonparametric function.

1. Introduction

A panel data set is one that follows a given sample of individuals over time, and thus provides multiple observations on each individual in the sample. Panel data involve two dimensions: a cross-sectional dimension and a time-series dimension. Such two-dimensional data enable researchers to analyze complex models and extract information and inferences which may not be possible using pure time-series data or cross-sectional data. With the increased availability, the panel data analysis is becoming more popular in recent years. Arellano (2003), Hsiao (2003), and Baltagi (2005) provided some excellent overviews on parametric panel data model analysis. To relax the strong assumptions in parametric panel data models, the econometricians and statisticians have also worked on nonparametric and semiparametric panel data models recently. For instance, Henderson et al. (2008), Su and Ullah (2011), and Wei and Wu (2009) considered the fixed effects nonparametric panel data models. Henderson and Ullah (2005), and Su and Ullah (2007) considered the random effects nonparametric panel data models. Baltagi and Li (2002), Li and Stengos (1996), Su and Ullah (2006), and Zhang et al. (2011) considered the partially linear panel data models with fixed effects, among others.

Consider the following nonparametric fixed effects panel data model,

$$y_{it} = \mu_i + g(x_{it}) + v_{it}, \quad i = 1, \ldots, n, \; t = 1, \ldots, T, \tag{1.1}$$

where $\{y_{it}\}$ are response variables, $\{\mu_i\}$ are unobserved individual effects, $g(\cdot)$ is an unknown smooth function, $\{x_{it}\}$ are explanatory variables in $[0, 1]$, and $\{v_{it}\}$ are random errors with zero mean. In addition, T is the time series length, n is the cross section size, and we assume that $\{\mu_i\}$ are independent and identically distributed (i.i.d.) random variables with zero mean and finite variance $\sigma_u^2 > 0$.

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We allow for that the individual effects $\{\mu_i\}$ are correlated with the explanatory variables $\{x_{it}\}$ in an unknown correlation structure. This leads to model (1.1) as a fixed effects model. For the purpose of identification, we assume that $\sum_{i=1}^{n}\mu_i = 0$ throughout this paper. Alternatively, we say model (1.1) is a random effects model when $\{\mu_i\}$ are uncorrelated with $\{x_{it}\}$.

One important problem in nonparametric and semiparametric regression is to construct the simultaneous confidence band (SCB) for the nonparametric function g. Despite an increasing body of literature in this area, little attention has been paid to constructing SCB for model (1.1). This is certainly not due to a lack of interesting applications, but mainly due to the considerable difficulties in formulating SCB for panel data models and in establishing their theoretical properties. Inspired by this, we propose to establish the asymptotic distributions of the normalized maximum deviation of the estimated nonparametric function from the true nonparametric function. The proposed results will then be applied to construct SCB for the nonparametric function and to make inference for the constructed SCB. The remainder of the paper is organized as follows. In Section 2, we present the main results. Specifically, we will propose the estimation procedure, establish the asymptotic properties, construct SCB for the nonparametric function, and propose a cross validation method for the optimal bandwidth selection. We then conclude the paper in Section 3 and present the simulation study and the technical proofs in the supplementary materials (Appendix).

2. Main results

2.1. Estimation procedure

For ease of notation, let $\boldsymbol{Y} = (y_{11}, \ldots, y_{1T}, \ldots, y_{n1}, \ldots, y_{nT})^{\mathrm{T}}$, $\boldsymbol{g} = (g(x_{11}), \ldots, g(x_{1T}), \ldots, g(x_{n1}), \ldots, g(x_{nT}))^{\mathrm{T}}$, $\boldsymbol{v} = (v_{11}, \ldots, v_{1T}, \ldots, v_{n1}, \ldots, v_{nT})^{\mathrm{T}}$ and $\boldsymbol{\mu}_0 = (\mu_1, \ldots, \mu_n)^{\mathrm{T}}$. Then model (1.1) has the following matrix form,

$$\boldsymbol{Y} = (I_n \otimes e_T)\boldsymbol{\mu}_0 + \boldsymbol{g} + \boldsymbol{v}, \tag{2.1}$$

where I_n is an $n \times n$ identity matrix, e_T is a T-dimensional column vector with all elements being 1, and $\otimes$ denotes the Kronecker product. Furthermore, by the identification assumption $\sum_{i=1}^{n}\mu_i = 0$, we have $\mu_1 = -\sum_{i=2}^{n}\mu_i$. Define the $(nT) \times (n-1)$ matrix $\boldsymbol{Z} = (z_{11}, \ldots, z_{1T}, \ldots, z_{n1}, \ldots, z_{nT})^{\mathrm{T}} = [-e_{n-1}, I_{n-1}]^{\mathrm{T}} \otimes e_T$ and $\boldsymbol{\mu} = (\mu_2, \ldots, \mu_n)^{\mathrm{T}}$, where $\{z_{it}\}$ are column vectors of size $n-1$. We rewrite (2.1) as

$$\boldsymbol{Y} = \boldsymbol{Z}\boldsymbol{\mu} + \boldsymbol{g} + \boldsymbol{v}. \tag{2.2}$$

Note that model (2.2) is a partially linear model with the unknown parameter vector $\boldsymbol{\mu}$ and the nonparametric function $g(\cdot)$. Thus, the profile least-squares method can be used to estimate $\boldsymbol{\mu}$ and $g(\cdot)$. By assuming $\boldsymbol{\mu}$ is known, model (2.2) reduces to a nonparametric regression model

$$y_{it} - z_{it}^{\mathrm{T}}\boldsymbol{\mu} = g(x_{it}) + v_{it}, \quad i = 1, \ldots, n,\ t = 1, \ldots, T. \tag{2.3}$$

There are different methods available for estimating the nonparametric function g. For simplicity, we apply the local polynomial method to estimate it in what follows. Specifically, for x_{it} close to $x \in [0, 1]$, we take the following local linear approximation,

$$g(x_{it}) \approx g(x) + g'(x)(x_{it} - x).$$

Given the $\boldsymbol{\mu}$ value, $\boldsymbol{\theta}(x) = (g(x), g'(x))^{\mathrm{T}}$ is then estimated by minimizing

$$\sum_{i=1}^{n}\sum_{t=1}^{T}\Big(y_{it} - z_{it}^{\mathrm{T}}\boldsymbol{\mu} - g(x) - g'(x)(x_{it} - x)\Big)^2 K_h(x_{it} - x), \tag{2.4}$$

where $K_h(\cdot) = K(\cdot/h)/h$ with $K(\cdot)$ being a kernel function and $h = h(n)$ being the bandwidth. Let

$$\boldsymbol{D}_x = \begin{pmatrix} 1 & \cdots & 1 & \cdots & 1 & \cdots & 1 \\ x_{11} - x & \cdots & x_{1T} - x & \cdots & x_{n1} - x & \cdots & x_{nT} - x \end{pmatrix}^{\mathrm{T}}$$

and $\boldsymbol{W}_x = \mathrm{diag}\Big(K_h(x_{11} - x), \ldots, K_h(x_{1T} - x), \ldots, K_h(x_{n1} - x), \ldots, K_h(x_{nT} - x)\Big)$ be an $(nT) \times (nT)$ diagonal matrix. Then the objective function (2.4) becomes

$$\Big(\boldsymbol{Y} - \boldsymbol{D}_x\theta(x) - \boldsymbol{Z}\boldsymbol{\mu}\Big)^{\mathrm{T}}\boldsymbol{W}_x\Big(\boldsymbol{Y} - \boldsymbol{D}_x\theta(x) - \boldsymbol{Z}\boldsymbol{\mu}\Big). \tag{2.5}$$

By minimizing (2.5), we have the solution of $\boldsymbol{\theta}(x)$ as

$$\widetilde{\boldsymbol{\theta}}(x, \boldsymbol{\mu}) = (\boldsymbol{D}_x^{\mathrm{T}}\boldsymbol{W}_x\boldsymbol{D}_x)^{-1}\boldsymbol{D}_x^{\mathrm{T}}\boldsymbol{W}_x(\boldsymbol{Y} - \boldsymbol{Z}\boldsymbol{\mu}). \tag{2.6}$$

In particular, the estimator of $g(x)$ is given as

$$\widetilde{g}(x, \boldsymbol{\mu}) = (1, 0)(\boldsymbol{D}_x^{\mathrm{T}}\boldsymbol{W}_x\boldsymbol{D}_x)^{-1}\boldsymbol{D}_x^{\mathrm{T}}\boldsymbol{W}_x(\boldsymbol{Y} - \boldsymbol{Z}\boldsymbol{\mu}), \tag{2.7}$$

where $(1, 0)$ is a row vector of size 2. Note that $\widetilde{g}(x, \boldsymbol{\mu})$ depends on the unknown fixed effects $\boldsymbol{\mu}$. We consider removing $\boldsymbol{\mu}$ by a least-squares dummy variable model as in parametric panel data model analysis. Specifically, we solve the following optimization problem to estimate $\boldsymbol{\mu}$,

$$\begin{aligned} &\min_{\boldsymbol{\mu}}\Big(\boldsymbol{Y} - \widetilde{\boldsymbol{g}}_{\mu} - \boldsymbol{Z}\boldsymbol{\mu}\Big)^{\mathrm{T}}\Big(\boldsymbol{Y} - \widetilde{\boldsymbol{g}}_{\mu} - \boldsymbol{Z}\boldsymbol{\mu}\Big) \\ &\quad = \min_{\boldsymbol{\mu}}\Big(\boldsymbol{Y}^* - \boldsymbol{Z}^*\boldsymbol{\mu}\Big)^{\mathrm{T}}\Big(\boldsymbol{Y}^* - \boldsymbol{Z}^*\boldsymbol{\mu}\Big), \end{aligned} \tag{2.8}$$

where $\widetilde{\boldsymbol{g}}_{\mu} = (\widetilde{g}(x_{11}, \boldsymbol{\mu}), \ldots, \widetilde{g}(x_{1T}, \boldsymbol{\mu}), \ldots, \widetilde{g}(x_{n1}, \boldsymbol{\mu}), \ldots, \widetilde{g}(x_{nT}, \boldsymbol{\mu}))^{\mathrm{T}} = \boldsymbol{S}(\boldsymbol{Y} - \boldsymbol{Z}\boldsymbol{\mu})$, $\boldsymbol{Y}^* = (I_{nT} - \boldsymbol{S})\boldsymbol{Y}$, $\boldsymbol{Z}^* = (I_{nT} - \boldsymbol{S})\boldsymbol{Z}$, and $\boldsymbol{S}$ is an $(nT) \times (nT)$ matrix of form

$$\boldsymbol{S} = \begin{pmatrix} (1, 0)(\boldsymbol{D}_{x_{11}}^{\mathrm{T}}\boldsymbol{W}_{x_{11}}\boldsymbol{D}_{x_{11}})^{-1}\boldsymbol{D}_{x_{11}}^{\mathrm{T}}\boldsymbol{W}_{x_{11}} \\ (1, 0)(\boldsymbol{D}_{x_{12}}^{\mathrm{T}}\boldsymbol{W}_{x_{12}}\boldsymbol{D}_{x_{12}})^{-1}\boldsymbol{D}_{x_{12}}^{\mathrm{T}}\boldsymbol{W}_{x_{12}} \\ \cdots \\ (1, 0)(\boldsymbol{D}_{x_{nT}}^{\mathrm{T}}\boldsymbol{W}_{x_{nT}}\boldsymbol{D}_{x_{nT}})^{-1}\boldsymbol{D}_{x_{nT}}^{\mathrm{T}}\boldsymbol{W}_{x_{nT}} \end{pmatrix}.$$

The resulting profile least-squares estimator of $\boldsymbol{\mu}$ is

$$\hat{\boldsymbol{\mu}} = [(\boldsymbol{Z}^*)^{\mathrm{T}}\boldsymbol{Z}^*]^{-1}(\boldsymbol{Z}^*)^{\mathrm{T}}\boldsymbol{Y}^* = (\boldsymbol{Z}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{Z})^{-1}\boldsymbol{Z}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{Y}, \tag{2.9}$$

where $\boldsymbol{Q} = (I_{nT} - \boldsymbol{S})^{\mathrm{T}}(I_{nT} - \boldsymbol{S})$. Given $\hat{\boldsymbol{\mu}}$, the estimator of μ_1 is $\hat{\mu}_1 = -\sum_{i=2}^{n}\hat{\mu}_i$. Finally, by (2.7) and (2.9) we have the following estimator of $g(\cdot)$,

$$\begin{aligned} \hat{g}(x) &= (1, 0)(\boldsymbol{D}_x^{\mathrm{T}}\boldsymbol{W}_x\boldsymbol{D}_x)^{-1}\boldsymbol{D}_x^{\mathrm{T}}\boldsymbol{W}_x(\boldsymbol{Y} - \boldsymbol{Z}\hat{\boldsymbol{\mu}}) \\ &= (1, 0)(\boldsymbol{D}_x^{\mathrm{T}}\boldsymbol{W}_x\boldsymbol{D}_x)^{-1}\boldsymbol{D}_x^{\mathrm{T}}\boldsymbol{W}_x\boldsymbol{M}\boldsymbol{Y}, \end{aligned} \tag{2.10}$$

where $\boldsymbol{M} = I_{nT} - \boldsymbol{Z}(\boldsymbol{Z}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{Z})^{-1}\boldsymbol{Z}^{\mathrm{T}}\boldsymbol{Q}$ is an $(nT) \times (nT)$ matrix such that $\boldsymbol{M}\boldsymbol{Z} = 0$.

2.2. Asymptotic properties

Here and in the sequel, define $\kappa_j = \int u^j K(u)du$ and $\nu_j = \int u^j K^2(u)du$ for $j = 0, 1, 2$. We also denote by $\mathcal{X} = \{x_{it},\ 1 \le i \le n,\ 1 \le t \le T\}$ the observed covariates. To establish the asymptotic results, we need the following conditions.

C1 Assume that $(\mu_i, \boldsymbol{x}_i, \boldsymbol{v}_i)$, $i = 1, \ldots, n$, are i.i.d., where $\boldsymbol{v}_i = (v_{i1}, \ldots, v_{iT})^{\mathrm{T}}$ and $\boldsymbol{x}_i = (x_{i1}, \ldots, x_{iT})^{\mathrm{T}}$. Furthermore, $E(|v_{it}|^{2+\delta}) < \infty$ for some $\delta > 0$, and let $\sigma^2(x) = \mathrm{Var}(v_{it}^2 | x_{it} = x)$ where $\sigma^2(x)$ is uniformly bounded, and $0 < c_1 \le \sigma^2(x) \le c_2 < \infty$.

C2 The function $g(\cdot)$ has continuous derivatives on [0, 1] up to the second order.

C3 Let $f(x) = \sum_{t=1}^T f_t(x)$, where $f_t(\cdot)$ denote the density function of x_{it}, and assume that $f_t(\cdot)$ is continuous and positive on the interval [0, 1] for each $t = 1, \ldots, T$. Furthermore, let $\widetilde{v}_{it} = v_{it} - \frac{1}{T}\sum_{s=1}^T v_{is}$, $\sigma_t^2(x) = E[\widetilde{v}_{it}^2 | x_{it} = x]$, and $\overline{\sigma}^2(x) = \sum_{t=1}^T \sigma_t^2(x) f_t(x)$.

C4 The kernel function $K(\cdot)$ is a symmetric density function, and is absolutely continuous on its support set $[-A, A]$. Furthermore, $K(A) = 0$, $K(u)$ is absolutely continuous and $K^2(u)$, $(K'(u))^2$ are integrable on the $(-\infty, \infty)$.

C5 The bandwidth h satisfies that $nh^3/\log n \to \infty$ and $nh^5 \log n \to 0$ as $n \to \infty$.

Theorem 1. *Assume that conditions* (C1)–(C5) *hold. Let* $b(x) = h^2\kappa_2 g''(x)/2$ *and* $\Sigma_{g(x)} = \nu_0 \overline{\sigma}^2(x) f^{-2}(x)$. *Then uniformly for* $x \in [0, 1]$, *we have*

$$\sqrt{nh}\Big\{\hat{g}(x) - g(x) - b(x)\Big\} \xrightarrow{L} N\Big(0, \Sigma_{g(x)}\Big), \tag{2.11}$$

where "$\xrightarrow{L}$*" denotes convergence in distribution.*

Theorem 2. *Assume that conditions* (C1)–(C5) *hold and* $h = O(n^{-\rho})$ *for* $1/5 \le \rho < 1/3$. *Then for all* $x \in [0, 1]$, *we have*

$$P\left\{(-2\log h)^{1/2}\left(\sup_{x\in[0,1]}\left|\left(nh\Sigma_{g(x)}^{-1}\right)^{1/2}\Big(\hat{g}(x) - g(x) - b(x)\Big)\right| - d_n\right) < z\right\} \longrightarrow \exp\Big(-2\exp(-z)\Big),$$

where $d_n = (-2\log h)^{1/2} + \frac{1}{(-2\log h)^{1/2}} \log\left\{\frac{1}{4\nu_0\pi}\int (K'(t))^2 dt\right\}$.

The asymptotic normality of $\hat{g}(x)$ in Theorem 1 is similar to the result in Su and Ullah (2006) in the framework of partially linear panel data models. Theorem 2 gives the asymptotic distribution of the maximum absolute deviation between the estimated $\hat{g}(\cdot)$ and the true $g(\cdot)$. It is worth mentioning that if the supremum in Theorem 2 is taken on an interval $[c_1, c_2]$ instead of [0, 1], Theorem 2 still holds under certain conditions by transformation. The resulting asymptotic distribution is

$$P\left\{(-2\log\{h/(c_2 - c_1)\})^{1/2}\left(\sup_{x\in[c_1,c_2]}\left|\left(nh\Sigma_{g(x)}^{-1}\right)^{1/2} \times \Big(\hat{g}(x) - g(x) - b(x)\Big)\right| - \widetilde{d}_n\right) < z\right\} \longrightarrow \exp\Big(-2\exp(-z)\Big),$$

where $\widetilde{d}_n$ is the d_n in Theorem 2 with h being replaced by $h/(c_2-c_1)$.

2.3. SCB for the nonparametric function

Noting that the bias and variance of $\hat{g}(\cdot)$ involve the unknown quantities, we cannot apply Theorem 2 directly to construct SCB for $g(\cdot)$. By Theorem 1, the asymptotic bias of $\hat{g}(x)$ is $(h^2\kappa_2/2)g''(x)(1 + o_P(1))$. We thus estimate the bias by $\widehat{\mathrm{bias}}(\hat{g}(x)) = h^2\kappa_2 \hat{g}''(x)/2$. The estimator $\hat{g}''(x)$ of $g''(x)$ is obtained by using local cubic fit with an appropriate pilot bandwidth $h_* = O(n^{-1/7})$, which is optimal for estimating $g''(x)$ and can be chosen by the residual squares criterion proposed in Fan and Gijbels (1996).

Now we estimate the asymptotic variance of $\hat{g}(x)$. For simplicity, suppose that v_{it} are i.i.d. for all i and t. Then by the proof of Theorem 1, we have

$$\mathrm{Var}\{\hat{g}(x)|\mathcal{X}\} = (1, 0)(\boldsymbol{D}_x^{\mathrm{T}}\boldsymbol{W}_x\boldsymbol{D}_x)^{-1}(\boldsymbol{D}_x^{\mathrm{T}}\boldsymbol{W}_x\boldsymbol{M}\Phi\boldsymbol{M}\boldsymbol{W}_x\boldsymbol{D}_x) \times (\boldsymbol{D}_x^{\mathrm{T}}\boldsymbol{W}_x\boldsymbol{D}_x)^{-1}(1, 0)^{\mathrm{T}},$$

where $\Phi = \mathrm{diag}(\sigma^2(x_{11}), \ldots, \sigma^2(x_{1T}), \ldots, \sigma^2(x_{n1}), \ldots, \sigma^2(x_{nT}))$. By the approximate local homoscedasticity, we can estimate the asymptotic variance of $\hat{g}(x)$ by

$$\widehat{\mathrm{Var}}\{\hat{g}(x)|\mathcal{X}\} = (1, 0)(\boldsymbol{D}_x^{\mathrm{T}}\boldsymbol{W}_x\boldsymbol{D}_x)^{-1}(\boldsymbol{D}_x^{\mathrm{T}}\boldsymbol{W}_x\boldsymbol{M}\boldsymbol{W}_x\boldsymbol{D}_x) \times (\boldsymbol{D}_x^{\mathrm{T}}\boldsymbol{W}_x\boldsymbol{D}_x)^{-1}(1, 0)^{\mathrm{T}}\sigma^2(x).$$

Let $\hat{\boldsymbol{v}} = \boldsymbol{Y} - \hat{\boldsymbol{g}} - \boldsymbol{Z}\hat{\boldsymbol{\mu}} = (\hat{v}_{11}, \ldots, \hat{v}_{1T}, \ldots, \hat{v}_{n1}, \ldots, \hat{v}_{nT})^{\mathrm{T}}$ be the residuals vector. By (2.9) and (2.10), we have

$$\begin{aligned}\boldsymbol{Y} &= \hat{\boldsymbol{g}} + \boldsymbol{Z}\hat{\boldsymbol{\mu}} + \hat{\boldsymbol{v}} = \boldsymbol{SMY} + \boldsymbol{Z}(\boldsymbol{Z}^{\mathrm{T}}\boldsymbol{QZ})^{-1}\boldsymbol{Z}^{\mathrm{T}}\boldsymbol{QY} + \hat{\boldsymbol{v}} \\ &= \boldsymbol{SMY} + (I_{nT} - \boldsymbol{M})\boldsymbol{Y} + \hat{\boldsymbol{v}} = \hat{\boldsymbol{Y}} + \hat{\boldsymbol{v}},\end{aligned} \tag{2.12}$$

where $\hat{\boldsymbol{Y}} = [I_{nT} + (\boldsymbol{S} - I_{nT})\boldsymbol{M}]\boldsymbol{Y}$ are the fitted values and $\hat{\boldsymbol{v}} = (I_{nT} - \boldsymbol{S})\boldsymbol{MY}$. Finally, by the normalized weighted residual sum of squares, we estimate $\sigma^2(x)$ by

$$\hat{\sigma}^2(x) = \frac{\hat{\boldsymbol{v}}^{\mathrm{T}}\hat{\boldsymbol{v}}}{\mathrm{tr}(\boldsymbol{M}^{\mathrm{T}}\boldsymbol{QM})} = \frac{\boldsymbol{Y}^{\mathrm{T}}(\boldsymbol{M}^{\mathrm{T}}\boldsymbol{QM})\boldsymbol{Y}}{\mathrm{tr}(\boldsymbol{M}^{\mathrm{T}}\boldsymbol{QM})}.$$

Theorem 3. *Assume that* $g^{(3)}(\cdot)$ *is continuous on* [0, 1] *and the pilot bandwidth* h_* *is of order* $n^{-1/7}$. *Then under the conditions in Theorem 2, for all* $x \in [0, 1]$ *we have*

$$P\left\{(-2\log h)^{1/2}\left(\sup_{x\in[0,1]}\left|\frac{\hat{g}(x) - g(x) - \widehat{\mathrm{bias}}(\hat{g}(x))}{[\widehat{\mathrm{Var}}\{\hat{g}(x)|\mathcal{X}\}]^{1/2}}\right| - d_n\right) < z\right\} \to \exp\Big(-2\exp(-z)\Big),$$

where d_n *is defined in Theorem 2.*

By Theorem 3, we can construct the $(1-\alpha) \times 100\%$ SCB of the nonparametric function as

$$\left(\hat{g}(x) - \widehat{\mathrm{bias}}(\hat{g}(x)) \pm \Delta_{1,\alpha}(x)\right),$$

where $\Delta_{1,\alpha}(x) = \Big(d_n + [\log 2 - \log\{-\log(1-\alpha)\}](-2\log h)^{-1/2}\Big)[\widehat{\mathrm{Var}}\{\hat{g}(x)|\mathcal{X}\}]^{1/2}$.

2.4. Bootstrap procedure

This subsection introduces a bootstrap procedure to construct SCB for $g(\cdot)$. Let

$$T = \sup_{x\in[0,1]} \frac{|\hat{g}(x) - g(x)|}{\{\mathrm{Var}(\hat{g}(x)|\mathcal{X})\}^{1/2}}.$$

Suppose that the upper α quantile of T is c_α. If both c_α and $\mathrm{Var}(\hat{g}(x)|\mathcal{X})$ are known, the confidence band of $g(\cdot)$ on the interval [0,1] would be

$$\hat{g}(x) \pm \{\mathrm{Var}(\hat{g}(x)|\mathcal{X})\}^{1/2}c_\alpha.$$

However, c_α and $\mathrm{Var}(\hat{g}(x)|\mathcal{X})$ are usually unknown in practice. Suppose that we have the estimators $\hat{c}_\alpha$ and $\mathrm{Var}^*(\hat{g}(x)|\mathcal{X})$ of c_α and $\mathrm{Var}(\hat{g}(x)|\mathcal{X})$, respectively. Then we can obtain the $(1-\alpha) \times 100\%$ confidence band of $g(\cdot)$ as follows

$$\hat{g}(x) \pm \{\mathrm{Var}^*(\hat{g}(x)|\mathcal{X})\}^{1/2}\hat{c}_\alpha.$$

In what follows, we apply the bootstrap methods in Goncalves and Kilian (2004) and Su and Chen (forthcoming) to estimate c_α and $\mathrm{Var}(\hat{g}(x)|\mathcal{X})$. The proposed algorithm is as follows:

(1) By (2.12), obtain the residuals vector $\hat{\boldsymbol{v}} = (I_{nT} - \boldsymbol{S})\boldsymbol{MY}$, where $\hat{\boldsymbol{v}} = (\hat{v}_{11}, \ldots, \hat{v}_{1T}, \ldots, \hat{v}_{n1}, \ldots, \hat{v}_{nT})^{\mathrm{T}}$.
(2) For each $i = 1, \ldots, n$ and $t = 1, \ldots, T$, obtain the bootstrap error $v_{it}^* = \hat{v}_{it}\varepsilon_{it}$, where ε_{it} are i.i.d. $N(0, 1)$ across i and t. Generate the bootstrap sample member y_{it}^* by $y_{it}^* = \hat{y}_{it} + v_{it}^*$ for $i = 1, \ldots, n$ and $t = 1, \ldots, T$, where $\hat{y}_{it}$ is fitted values of y_{it} defined in (2.12).
(3) Given the bootstrap resample $\{(y_{it}^*, x_{it}),\ i = 1, \ldots, n,\ t = 1, \ldots, T\}$, obtain the estimator of $g(\cdot)$, and denote the resulting estimate by $\hat{g}^*(\cdot)$.
(4) Repeat (2)–(3) m times to get a size m bootstrap sample of $\hat{g}(\cdot)$: $\hat{g}_k^*(\cdot),\ k = 1, \ldots, m$. The estimator $\mathrm{Var}^*(\hat{g}(\cdot))$ of $\mathrm{Var}(\hat{g}(\cdot))$ is taken as the sample variance of $\hat{g}_k^*(\cdot),\ k = 1, \ldots, m$.
(5) Repeat (2)–(3) M times to get a bootstrap sample of size M for $\hat{g}(\cdot) : \hat{g}_k^*(\cdot),\ k = 1, \ldots, M$. Compute

$$T_k^* = \sup_{x\in[0,1]} \frac{|\hat{g}_k^*(x) - \hat{g}(x)|}{\{\mathrm{Var}^*(\hat{g}(x)|\mathcal{X})\}^{1/2}}, \quad k = 1, \ldots, M.$$

(6) Use the upper α percentile of $T_k^*,\ k = 1, \ldots, M$, to estimate the upper α quantile c_α of T.

3. Conclusion

This paper considers the nonparametric fixed effects panel data models. We first transform the nonparametric fixed effects panel data models into the partially linear models. We then obtain the estimator of the nonparametric function and remove the fixed effects using the profile least-squares method. The asymptotic distributions of the normalized maximum deviation of the estimated nonparametric function from the true nonparametric function are also derived. The proposed results can be used to construct SCB for the nonparametric function, and to construct the test statistics for addressing graphical questions related to the nonparametric function. For instance, if the constructed $(1-\alpha)\times 100\%$ SCB for the nonparametric function over the set [0, 1] does not contain any linear function, then it will be evident that the link function $g(\cdot)$ is nonlinear. That is, a graphical representation of the constructed SCB will suggest when to reject the null hypothesis that $g(\cdot)$ is a linear function. Finally, we note that the methods proposed in this paper can be readily extended to various semiparametric panel data models with fixed effects.

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Appendix. Supplementary data

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.econlet.2013.02.037.

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